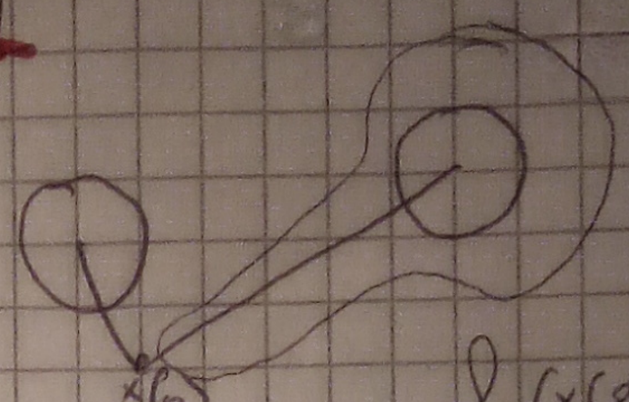
Firenze 06/18

SELECTED TOPICS IN QML

★ $\vec{X} \in \mathbb{R}^{n \times d}$ $\vec{y} \in \mathbb{R}^{d}$

- SUP. VS UNSUPERVISED →
- ROWS OF $\vec{X}$ as samples



KNN
NC

$h(x(0)) = ?$ $h: \mathbb{R}^d \to [K]$

★ What is a Q.A. ? ⇒ $\exists h$: given $U_i: |\psi\rangle_{IN} \to |\psi_i\rangle_{OUTPUT}$ $\varepsilon, \eta, \ldots > 0$ ⇒

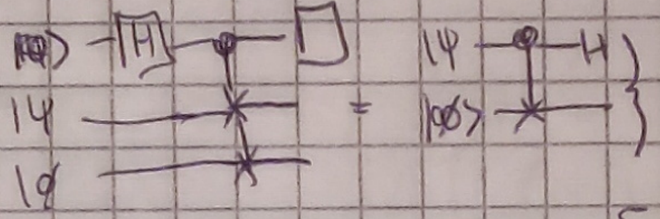
QMAM →
OPEN $U_1 \ldots U_k$ = PE →
H.S. →
AA, AE →

$\exists\, U: |\psi\rangle \to |\tilde{\phi}\rangle$ s.t.

$\| |\tilde{\phi}\rangle - |\phi\rangle \| < \varepsilon \wedge \| \overline{|\psi\rangle} - |\psi\rangle \| < \eta$

w.p. in time $O(\log Nd)$

★ SWAPS:



$$p(0) = \frac{1}{2}\left(1 - |\langle\psi|\phi\rangle|^2\right) \qquad \text{w.p. } O\left(\frac{\overline{T(U_\psi) + T(U_\phi)}}{\varepsilon^2}\right) \quad = \langle\phi\rangle \text{ DISTANCE}$$

★ SWAPS 4 N.C.

$$\text{QRAM} \begin{cases} |\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|x(0)\rangle + \frac{1}{N}\sum_{i=0}^{N}|i\rangle|x(0)\rangle\right) \\ |\phi\rangle = \frac{1}{z}\left(\frac{\|x(0)\|}{M}|0\rangle - \frac{1}{N^2}\sum_{i=0}^{N-1}\|x(i)\||i\rangle\right) \end{cases} \qquad z = \|x(0)\|^2 + \frac{1}{M}\sum_{i=0}^{N}\|x(i)\|^2$$

$$d\left(x(0), \frac{1}{\pi}\sum_{i=0}^{M} x(i)\right) = z\, p(0)$$

★ QFDC: $X_1 \in \mathbb{R}^{N_1 \times dL} \ldots X_k \in \mathbb{R}^{N_k * dL}$

$$F_k(x(0)) = \frac{\|X_* - X_0\|}{2\|X_0\| + \|X_k\|} \to N_k$$

$$h(x(0)) = \min_k \{F_k(x(0))\}$$

$X_0 = x(0)$ T_k TIMES

- WE KNOW $N_k, \|X_0\|, \|X_k\|$

AVERAGE SQUARED DISTANCE

$$|\psi\rangle = \frac{1}{\sqrt{N_k}}\left(|0\rangle \sum_{i \in T_k}\|x(0)\||i\rangle|x(0)\rangle + |1\rangle \sum_{i \in \bar{T}_k}\|x(i)\||i\rangle|x(i)\rangle\right) \xrightarrow{H}$$

$$p(|1\rangle) = F_k(x(0))$$

SAMPLE & ESTIMATE BY RUNNING $O\left(\frac{1}{\eta^2}\right)$

★ QUANTUM LINEAR ALGEBRA : Let $M \in \mathbb{R}^{n \times d} = \sum_{i \in [r]} \sigma_i u_i v_i^T$ $\|\sigma\| < 1$ in QRAM. Then $\exists$ Q.A.

- $|z_\varepsilon\rangle$ s.t. $\| |\Pi x\rangle - |Mx\rangle \| < \varepsilon \left(\| |z\rangle \| \cdot \| \Pi \| \cdot \| x \| \right) \leq \eta$ $O\left(\frac{\kappa(\Pi)\mu(\Pi)}{(\eta)\varepsilon \to \log \frac{1}{\varepsilon}} \right)$
- $|z\rangle$ $\| \Pi^{-1} \| \cdot \| \Pi \| \cdot \| x \|$ $O\left(\frac{\mu(\Pi)\|x\|}{\delta\emptyset \| \Pi^{-1} x \rangle \|} \right)$
- $|M^+_{\leq \delta, \emptyset} M_{\leq \delta, \emptyset}\rangle \longrightarrow M_{\leq \delta} = \sum_{\sigma_i < \delta} \sigma_i v_i w_i^T$

PRODUCTS OF MATRICES $\Pi = \Pi_1 \Pi_2$

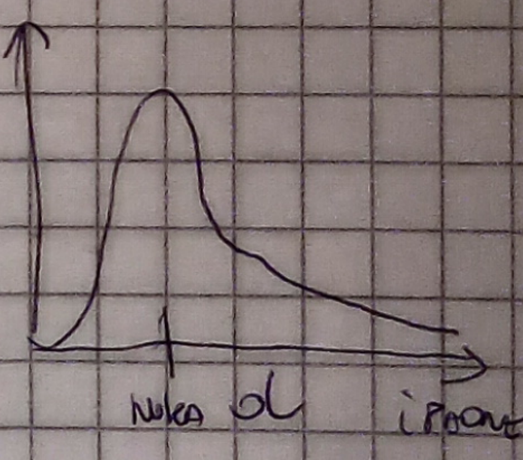
$$|\Pi x\rangle, |\Pi^{-1} x\rangle = O\left(\frac{\kappa(\Pi_1)\left(\mu(\Pi_1) + \mu(\Pi_2)\right)}{\log \frac{1}{\varepsilon}} \right)$$

$$|\Pi^+_{\leq \delta, \emptyset} M_{\leq \delta, \emptyset} x\rangle = O\left(\frac{\left(\mu(\Pi_1) + \mu(\Pi_2)\right)\|x\|}{\delta\emptyset \| \Pi^+_{\leq \delta, \emptyset} \Pi_{\leq \delta, \emptyset} \|} \right)$$

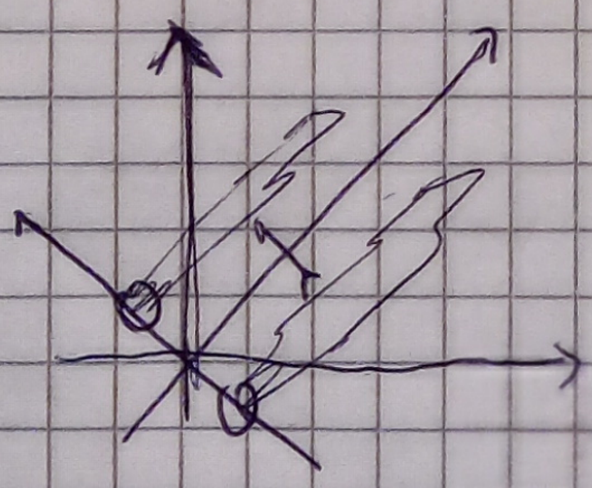
★ QUIZ : Course of dim.

N NOKIA ING : 80% ACCURACY

N IPHONE ING : ? ACCURACY ?



★ PCA



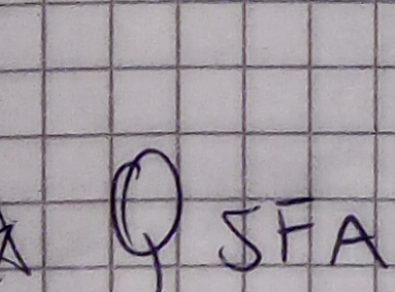
★ Q SFA $X \in \mathbb{R}^{n \times d}$ $\vec{y} \in [\mathbb{R}]^n$ $x(i) = [x_1(i) \dots x_d(i)]$

FIND $y(i) = [g_1(x(i)) \dots g_{k-1}(x(i))]$ s.t. $\forall s \in [k-1]$

$$\Delta_s = \frac{1}{a} \sum_{k=0}^{K} \sum_{\substack{i,i' \\ i<i'}}^{T_k} = \left(g_s(x(i)) - g_s(x(i')) \right)^2 \quad \text{min.}$$

(2/3)

- $\frac{1}{n} \sum_{k=0}^{K} \sum_{i=0}^{T_k} y_s(i) = 0$
- $\frac{1}{n} \sum_{k=0}^{K} \sum_{i=0}^{T_k} (y_s(i))^2 = 1$
- $\frac{1}{n} \sum_{k=0}^{K} \sum y_s(i) y_v(i)$ $\forall v < y$



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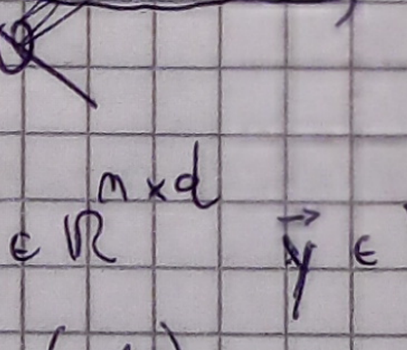


LINEAR CASE

$g_s(x(i)) = \langle w_s, x \rangle$

... For reasons... $\Delta_s = \frac{w_s A w_s}{w_s B w_s}$ = SOLUTION OF $Aw = \Lambda B w$

map SFA to this optimization problem

RAYLEIGH-QUOTIENT

$$\begin{bmatrix} A_x = \lambda x \\ Aw = \Lambda w \end{bmatrix}$$

$A = \dot{X}^T \dot{X}$ $\quad B = X^T X = U \Sigma U^T$

IN ORDER TO $B \rightarrow I$ FIND HOW $X^T \rightarrow Z^T$ s.t. $Z^T Z = I$ ... $Z = B^{-1/2} X^T$ ← SOLUTION

$$B^{-1/2} = U \cdot \Sigma^{-1/2} U$$

$$\dot{A} = \dot{Z}^T \dot{Z} = \sum_i^d \sigma_i w_i w_i^T$$

$(B^{-1/2} \cdot X^T)$ $\quad$ TAKE $K-1$ SMALLEST

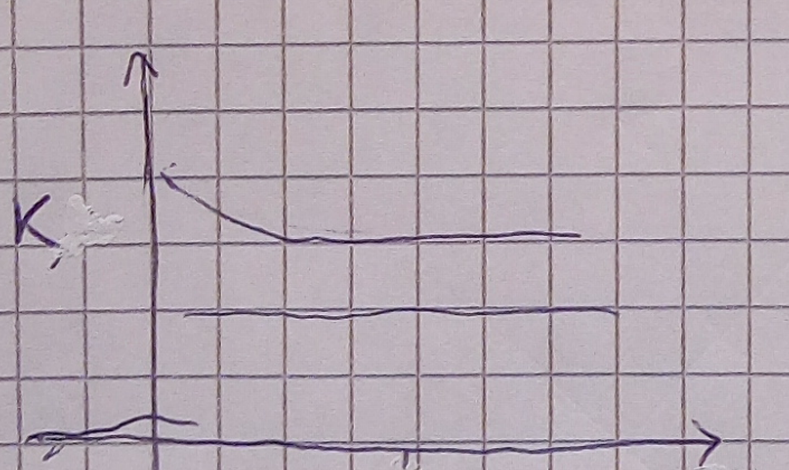
ALGO: WHITEN + PROJECT

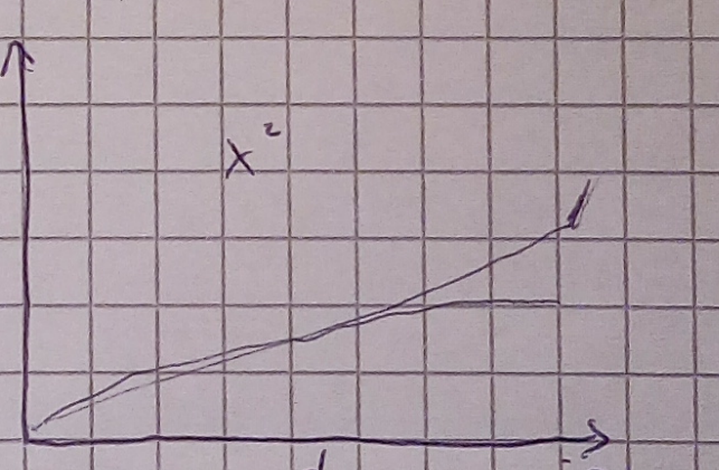
$$|X\rangle \rightarrow |z\rangle \rightarrow |Y\rangle = \frac{1}{N_x} \sum \|x(i)\| \, |i\rangle |x(i)\rangle \left(\rightarrow \frac{1}{N_z} \sum \|z(i)\| \, |i\rangle |z(i)\rangle \right)$$

$$\rightarrow \frac{1}{N_y} \sum \|y(i)\| \, |i\rangle |x(i)\rangle$$

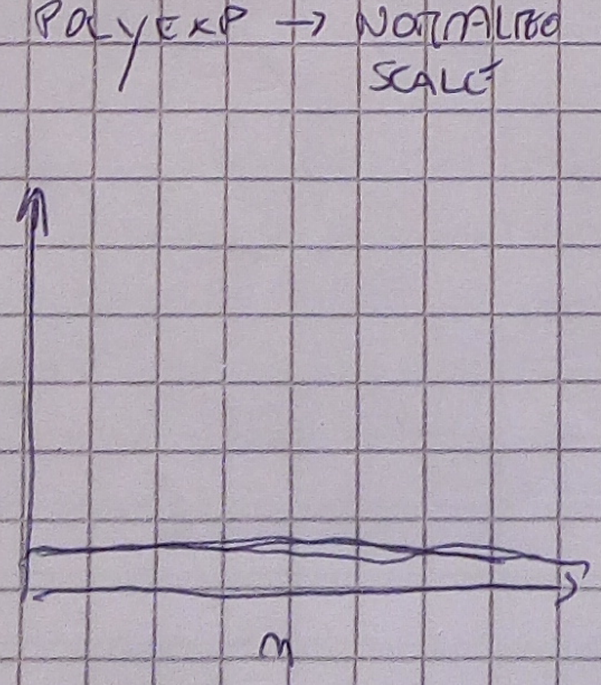
$$O\left(\left(\frac{\kappa(x) \mu(x)}{\log \frac{1}{\varepsilon}} + \frac{\kappa(x)\left(\mu(x) + \mu(\dot{x})\right)}{\delta, \emptyset} \right) \frac{\|z\|}{|A^+_{\delta,\emptyset} A_{\delta,\emptyset} z|} \cdot \log(Nd) \right)$$

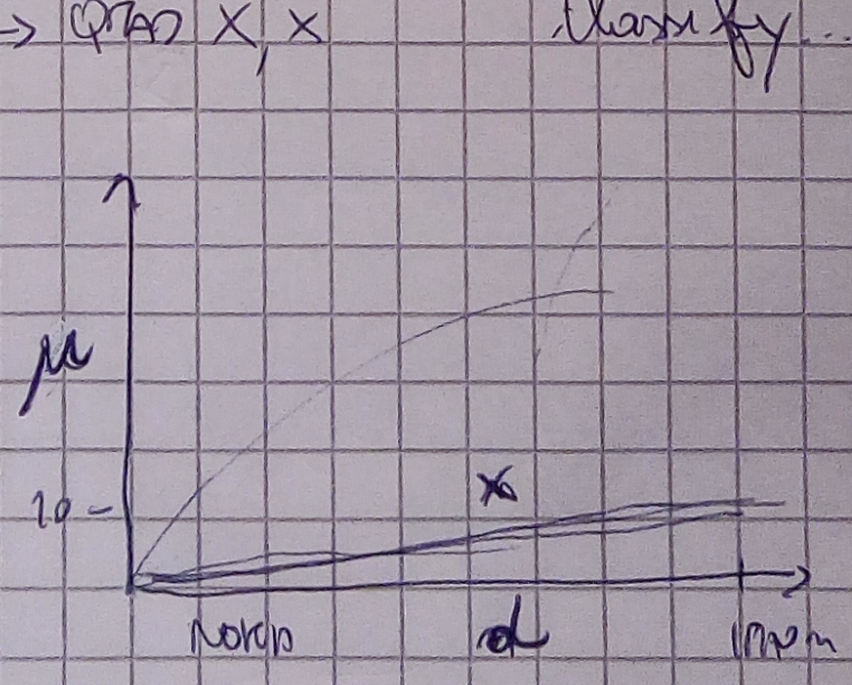
✱ MNIST





POLYEXP → NORMALIZED SCALE → $\dot{X}$ → GRAD $X, \dot{X}$ classify...





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