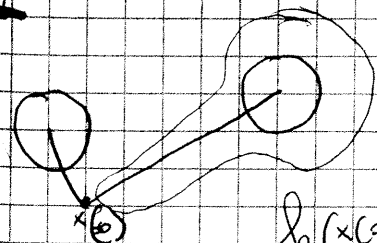
SELECTED TOPICS IN QML

Guinea 06/18



- $X \in \mathbb{R}^{n \times d}$ $\quad \vec{y} \in \mathbb{R}^{d}$

KNN
NC

- SUP. VS UNSUPERVISED →
- ROWS OF X AS samples

$h(x(0)) = ?$ $\quad h: \mathbb{R}^d \to [K]$

- What is a Q.A.? ⇒ Th: Given U_i $|\psi_{input}\rangle \to |\psi_{output}\rangle$ $\quad \varepsilon, \eta, \ldots > 0$ ⇒

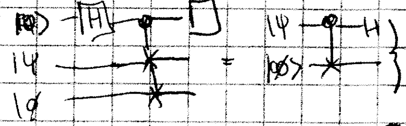
OFTEN $U_1 \ldots U_K$ = QRAM →, P.E →, H.S. →, AA, AE →

$\exists\ U\,|\psi\rangle \to |\tilde{\phi}\rangle$ s.t.

$\| |\tilde{\phi}\rangle - |\phi\rangle \| < \varepsilon \ \wedge\ \| |\tilde{\psi}\rangle - |\psi\rangle \| <$

w.p. in time $O(\log Nd)$

- SWAPS:



$$p(0) = \frac{1}{2}\left(1 - |\langle \psi | \phi \rangle|^2\right) \qquad O\left(\frac{T(U_\psi) + T(U_\phi)}{\varepsilon^2}\right) \quad = \text{COS DISTANCE}$$

- SWAPS 4 N.C.

QRAM

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|x(0)\rangle + \frac{1}{N}\sum_{i=0}^{N} |i\rangle|x(0)\rangle\right) \qquad Z = \|x(0)\|^2 + \frac{1}{N}\sum_{i=0}^{N} \|x(i)\|$$

$$|\phi\rangle = \frac{1}{Z}\left(\|x(0)\|\,|0\rangle + \frac{1}{N^2}\sum_{i=0}^{N} \|x(i)\|\,|i\rangle\right)$$

$$d\left(x(0), \frac{1}{n}\sum_{i=0}^{N} x(i)\right) = Z\,p(0)$$

- QFDC: $X_1 \in \mathbb{R}^{N_1 \times d} \ \ldots\ X_K \in \mathbb{R}^{N_K \times d}$

$$F_K(x(0)) = \frac{\|X_K - X_0\|}{2\left(\|X_0\| + \|X_K\|\right)} \to N_K$$

$$h(x(0)) = \min_K \{F_K(x(0))\}$$

$X_0 = x(0)$ T_K TIMES

AVERAGE SQUARED DISTANCE

- WE KNOW $N_K, \|X_0\|, \|X_K\|$

$$|\psi\rangle = \frac{1}{\sqrt{N_K}}\left(|0\rangle \sum_{i \in T_K} \|x(0)\|\,|i\rangle|x(0)\rangle + |1\rangle \sum_{i \in T_K} \|x(i)\|\,|i\rangle|x(0)\rangle\right) \xrightarrow{H}$$

$p(|1\rangle) = F_K(x(0))$ $\quad$ SAMPLE & ESTIMATE BY RUNNING $\quad O\left(\frac{1}{\eta^2}\right)$

QUANTUM LINEAR ALGEBRA Let $M \in \mathbb{R}^{n \times d} = \sum_{i \in [r]} \sigma_i v_i w_i^T$ $\|M\| < 1$ in QRAM. Then $\exists$ Q.A.

- $|z\rangle$ s.t. $\| |\bar{z}\rangle - |Mx\rangle \| < \varepsilon$ $\left(\| |\bar{z}\rangle - |Mx\rangle \| \right) <_m$ $O\left(\frac{\kappa(M)\mu(M)}{(\eta)\, \varepsilon \to \log \frac{1}{\varepsilon}}\right)$
- $|z\rangle$ $\| M^{-1} \quad M^{-1} \| <$
- $|M^{+}_{\leq \delta, \phi} M_{\leq \delta, \phi}\rangle$ $O\left(\frac{\mu(M)\|x\|}{\delta\phi \| |\dagger\rangle \|}\right)$

$\longrightarrow M_{\leq \delta} = \sum_{\sigma_i \leq \delta} \sigma_i v_i w_i^T$

PRODUCTS OF MATRICES $M = M_1 M_2$

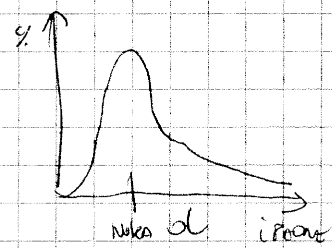
$$|Mx\rangle, |M^{-1}x\rangle = O\left(\frac{\kappa(M_1)\left(\mu(M_1) + \mu(M_2)\right)}{\log \frac{1}{\varepsilon}}\right)$$

$$|M^{+}_{\leq \delta, \phi} M_{\leq \delta, \phi} x\rangle = O\left(\frac{\left(\mu(M_1) + \mu(M_2)\right)\|x\|}{\delta\phi \| M^{+}_{\leq \delta, \phi} M_{\leq \delta, \phi} \|}\right)$$

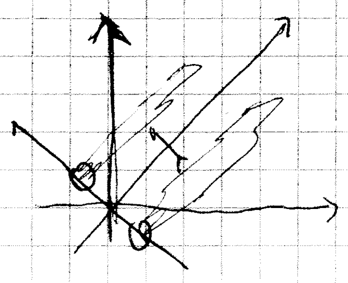
QUIZ Course of dim.

N NOKIA IMG : 80% ACCURACY

N IPHONE IMG : ? ACCURACY? $\to$



PCA



QSFA $X \in \mathbb{R}^{n \times d}$ $\vec{y} \in [K]^n$ $x(i) = [x_1(i) \dots x_d(i)]$

FIND $y(i) = [g_1(x(i)) \dots g_{K-1}(x(i))]$ s.t. $\forall\, s \in [K-1]$

$$\Delta_s = \frac{1}{a} \sum_{k=0}^{K} \sum_{\substack{i,i' \\ i<i'}}^{T_k} = \left(g_s(x(i)) - g_s(x(i'))\right)^2 \quad \min.$$

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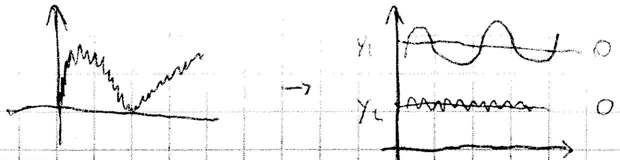
- $\frac{1}{m} \sum_{k=0}^{K} \sum_{i=0}^{T_k} y_s(i) = 0$
- $\frac{1}{m} \sum_{k=0}^{K} \sum_{i=0}^{T_k} (y_s(i))^2 = 1$
- $\frac{1}{m} \sum_{k=0}^{K} \sum y_s(i) y_v(i)$



LINEAR CASE

$\forall v < y$

$g_s(x(i)) = \langle w_s, x(i) \rangle$

... For reasons... map SFA to this optimization problem RAYLEIGHT-QUOTIENT

$$\Delta_s = \frac{w_s A w_s}{w_s B w_s}$$

w_s ... = SOLUTION OF

$$AW = \Lambda B W$$

$$\left[\begin{array}{l} A x = \lambda x \\ AW = \Lambda W \end{array}\right]$$

$$A = \dot{X}^T \dot{X} \qquad B = X^T X = U \Sigma U^T$$

IN ORDER TO

$B \rightarrow I$ FIND HOW $X^T \rightarrow Z^T$ s.t. $Z^T Z = I$ ... $Z = B^{-1/2} X^T$ ← SOLUTION

$$\underline{B^{-1/2} = U \Sigma^{-1/2} U}$$

$$\dot{A} = \dot{Z}^T \dot{Z} = \sum_i^d \alpha_i w_i w_i^T \qquad \text{TAKE} \quad K-1 \quad \text{SMALLEST}$$

$$\underline{\left(B^{-1/2} \dot{X}^T\right)}$$

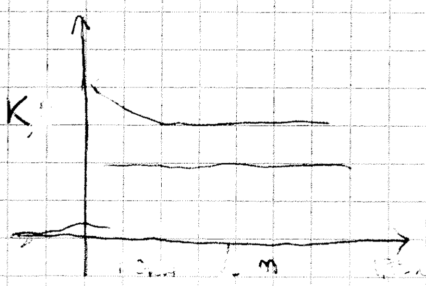
ALGO: WHITEN + PROJECT

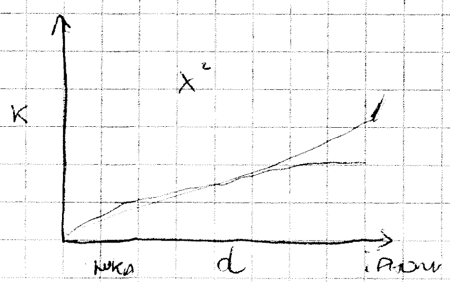
$$|X\rangle \rightarrow |Z\rangle \rightarrow |Y\rangle = \frac{1}{N_x} \sum \|x(i)\| \, |i\rangle |x(i)\rangle \left(\rightarrow \frac{1}{N_z} \sum \|z(i)\| \, i\rangle |z(i)\rangle \right)$$

$$\rightarrow \frac{1}{N_y} \sum \|y(i)\| \, |i\rangle |x(i)\rangle$$

$$O\left(\left(\frac{\kappa(x)\mu(x)}{\log \frac{1}{\varepsilon}} + \frac{\kappa(x)\left(\mu(X) + \mu(\dot{X})\right)}{\delta, \theta} \right) \frac{\|z\|}{\|A^+_{\delta,\theta} A_{\delta,\theta} z\|} \cdot \log(nd) \right)$$

MNIST





PAYEXP → NORMALIZED SCALE → $\dot{X}$ → QPAD $X, \dot{X}$ classify.